

CHAPTER 2

DAV CENTENARY PUBLIC SCHOOL, PASCHIM ENCLAVE, NEW DELHI-87

CLASS-XII

SESSION 2016-17

Inverse trigonometric function. Expected marks : 5 (1 + 4)

Expected No. of Questions : 2

One Mark Questions

Q1. Write the principle value of following.

a) $\cos^{-1}\cos\frac{7\pi}{6}$ b) $\sin^{-1}\sin\frac{3\pi}{5}$ c) $\cos^{-1}\left(\frac{-1}{2}\right)$ d) $\tan^{-1}(-1)$

e) $\sin^{-1}\tan\frac{3\pi}{4}$ f) $\cos^{-1}\cos\frac{-\pi}{4}$ g) $\sec^2(\tan^{-1}2)$ h) $\cot^{-1}\cot\frac{-\pi}{4}$

Q2. If $\sin\left\{\sin^{-1}\frac{1}{5} + \cos^{-1}x\right\} = 1$. Find x

Q3. Evaluate : a) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ b) $\cos^{-1}\cos\left(\frac{2\pi}{3}\right) + \sin^{-1}\sin\left(\frac{2\pi}{3}\right)$

Q4. Write one branch of $\tan^{-1}x$ other than principle branch.

Q5. Simplify the following expressions:

a) $\tan\frac{-\sqrt{1-\cos x}}{\sqrt{1+\cos x}}$ b) $\cot\frac{-ab+1}{a-b} + \cot\frac{-bc+1}{b-c} + \cot\frac{-ac+1}{a-c}$

c) $\tan\frac{-x}{y} \tan\frac{-x-y}{x+y}$ d) $\tan\frac{-x-1}{x+1} + \tan\frac{-x+1}{x-1}$

e) $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$ f) $\cos\left(2\tan^{-1}\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$

Four marks questions

Q1. Prove the following:

a) $\cos\left(\tan^{-1}\frac{-4}{3} + \sin^{-1}\frac{12}{13}\right) = \frac{63}{65}$ b) $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$

c) $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = 2(\cot^{-1}1 + \cot^{-1}2 + \cot^{-1}3)$

d) $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$ e) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}\frac{3}{5}$

f) $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2}$ g) $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{163}{16} = \pi$

Q2. Draw the graph of a) $f(x) = \tan^{-1} x$ b) $g(x) = \operatorname{cosec}^{-1} x$.

Q3. Simplify the following inverse trigonometric expressions:

a) $\cot^{-1}(\sqrt{1+x^2} + x)$

b) $\tan^{-1} \frac{\cos x}{1+\sin x}$

c) $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right)$

d) $\sin^{-1}\left(\frac{3x-4\sqrt{1-x^2}}{5}\right)$ e) $\sin^{-1}\frac{x}{\sqrt{9+x^2}}$. $\left(\quad \right)$

f) $\operatorname{cosec}^{-1}\left(\frac{\sqrt{4^2+x^2}}{x}\right)$ g) $\tan^{-1}\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}$ $\left(\quad \right)$

Q4. Solve for x :

a) $\tan(\cos^{-1}x) = \sin(\cot^{-1}\frac{1}{2})$

b) $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

c) $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$

d) $\tan^{-1}\frac{1+x}{1-x} = \frac{\pi}{4} + \tan^{-1}x$

e) $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

f) $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

Q5. Show that $\frac{1}{2}\tan^{-1}x = \cos^{-1}\left(\frac{\sqrt{1+x^2} + 1}{2\sqrt{1+x^2}}\right)$

Q6. Show that $4\tan^{-1}x = \tan^{-1}\frac{4x(1-x^2)}{1-6x^2+x^4}$

Q7. Show that $2\tan^{-1}\left(\sqrt{\frac{x-y}{x+y}} \tan\frac{\theta}{2}\right) = \cos^{-1}\left(\frac{y+x\cos\theta}{y+x\cos\theta}\right)$

Q8. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = z$. Show that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos z + \frac{y^2}{b^2} = \sin^2 z$.

Q9. If $y = \cot^{-1}\sqrt{\cos x} - \tan^{-1}\sqrt{\cos x}$. Prove that $\sin y = \tan^2\frac{x}{2}$