

# CHAPTER 1

## Relation, Function and Binary operations

### SESSION 2016-17

Expected marks : 5 (1 + 4)

Expected No. of Questions : 2

#### One Mark Questions

Q1. If  $f(x) = 2x-3$ . Write  $f^{-1}(5)$

Q2. Find the identity element of binary operation defined by  $(a,b) * (c,d) = (ac, b+d)$ ,  $a,b,c,d \in \mathbb{Z}$

Q3. If '\*' is defined on the set R of all real numbers by  $a * b = \sqrt{a^2 + b^2}$ , find the  $3 * 4$

Q4. If '\*' is defined on the set R of all real numbers by  $a * b = \frac{3ab}{7}$ , find the identity element in R for the binary operation \*.

Q5. Check whether a relation  $R = \{(a,b) : a < b^3, a,b \in \mathbb{N}\}$  is transitive or not. Justify

Q6. Find inverse of 5 in  $\mathbb{Q}_+$ , defined by  $\square$  binary operation \* as  $a * b = \frac{ab}{2} \forall a, b \in \mathbb{Q}^+$

Q7. Is the function  $f : \mathbb{N} \rightarrow \mathbb{N}$  given by  $f(1) = f(2) = 1$  and  $f(x) = x - 1$ , for every  $x > 2$  one-one? Justify

Q8. Check whether the relation R on the set  $A = \{1, 2, 3\}$  given by  $R = \{(1, 2), (2, 1)\}$  is transitive or not. Give reasons

Q9. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is given by  $f(x) = (3 - x^3)^{\frac{1}{3}}$ , then find  $f \circ f(x)$ .

Q10. If  $f(x) = 2x+5$  and  $g(x) = x - 1$ . Find  $f \circ g(2)$

Q11. If  $f(x) = |x|$  and  $g(x) = [x-1]$  where  $[.]$  denotes greatest integer function. Find  $f \circ g(-2.5)$

Q12. Write the number of one – one functions from  $\{1,2,3\}$  to itself.

Q13. Let \* be binary operation defined on N as  $a * b = \text{H.C.F}(a,b)$ . Find  $16 * 20$

Q14. Let R be relation defined on R as  $(a, b)$  iff  $1+ab > 0$ . Is R reflexive ?

Q15.  $R = \{(a,b) : a+b = 6, a,b \in \{1,2,3,4\}\}$ . Write range of R.

Q16. Write the domain of  $f(x) = \frac{1}{x^2-4}$ .

Q17. Write the range of  $f(x) = \frac{x-1}{|x-1|}$ .

Four marks questions

Q1. Let R be relation defined on  $N \times N$  as  $R = \{(a,b)R(c,d) \text{ iff } a+d = b+c\}$ . Show that R is an equivalence relation.

Q2. Let R be relation defined on  $R \times R$  as  $R = \{(a,b) : |a-b| \text{ is divisible by } 5\}$ . Show that R is an equivalence relation.

Q3. Show that  $f: R \rightarrow R$  defined as  $f(x) = 7-2x^3$  is bijective.

Q4. Show that  $f: R \rightarrow R$  defined as  $f(x) = \frac{3x+5}{2}$  is invertible. Hence find  $f^{-1}$ .

Q5. Show that  $f(1/x) = -f(x)$ , where  $f(x) = x^2 - x^{-2}$ .

Q6. Let  $f: N \rightarrow N$  be defined as  $f(x) = x + 1$ ,  $x$  is odd . Show that  $f$  is onto. Hence find  $f^{-1}$ .  
 $x - 1$ ,  $x$  is even

Q7. Show that  $a*b = a + b - ab$ ,  $a, b \in Q$  is commutative and associative.

Q8. Show that  $a*b = a + b$  if  $a + b < 6$  is binary operation where  $a, b \in \{0,1,2,3,4,5\}$   
 $a + b - 6$  if  $a + b > 5$ .

Hence prove that  $*$  is commutative and associative.

Also find identity element and all invertible elements.

Q9. Let  $A = \{1,2,4\}$  and let  $*$  be defined on A as  $a*b = \text{L.C.M}(a,b)$ . Is  $*$  binary?

Hence prove that  $*$  is commutative and associative.

Also find identity element and all invertible elements.

Q10. Let  $A = R - \{3\}$  and  $B = R - \{1\}$ . Let  $f: A \rightarrow B$  be defined as  $f(x) = \frac{x-2}{x-3}$ . Define  $g: B \rightarrow A$  such that  $f \circ g = \text{id}_B$  and  $g \circ f = \text{id}_A$ .

Q11. Show that  $f: R^+ \rightarrow [-5, \infty)$  defined as  $f(x) = 9x^2 + 6x - 5$  is invertible. Hence find  $f^{-1}$ .

( $R^+$  is set of non negative real numbers)